Low-Complexity Energy-Efficient Scheduling for Uplink OFDMA

Guowang Miao, Nageen Himayat, Geoffrey Ye Li, and Shilpa Talwar

***Abstract*—Energy-efficient wireless communication is very important for battery-constrained mobile devices. For mobile devices in a cellular system, uplink power consumption dominates the wireless power budget because of RF power requirements for reliable transmission over long distances. Our previous work in this area focused on optimizing energy efficiency by maximizing the instantaneous bits-per-Joule metric through iterative approaches, which resulted in significant energy savings for uplink cellular OFDMA transmissions. In this paper, we develop energy efficient schemes with significantly lower complexity when compared to iterative approaches, by considering time-averaged bits-per-Joule metrics. We consider an uplink OFDMA system where multiple users communicate to a central scheduler over frequency-selective channels with high energy efficiency. The scheduler allocates the system bandwidth among all users to optimize energy efficiency across the whole network. Using time-averaged metrics, we derive energy optimal techniques in "closed forms" for per-user link adaptation and resource scheduling across users. Simulation results show that the proposed schemes not only have low complexity but also perform close to the globally optimum solutions obtained through exhaustive search.**

***Index Terms*—Energy efficiency, OFDMA, bits per Joule, link adaptation, resource allocation.**

I. Introduction

Wireless communication systems have experienced tremendous growth in the past couple of decades. While this growth is expected to continue unabated, the continued success of wireless networks depends on their ability to efficiently utilize limited network resources to meet increasingly higher quality-of-service (QoS) requirements. While higher capacity wireless links are expected to meet the increasing QoS demand of multimedia applications, these high data rate links also result in increasing device power consumption. The slow improvement of battery technologies [2] has led to an exponentially increasing gap between the required and available battery capacity [3]. Additionally, shrinking device sizes further impose an ergonomic limit on battery capacity. Hence, energy efficiency is becoming increasingly important for wireless system design.

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As wireless is a shared medium, device energy efficiency is affected not only by the layers composing the point-to-point communication link, but also by the interaction between the links in the entire network. Hence, a systematic approach, including both transmission and multi-user resource management, is required for energy-efficient wireless communications.

Since the quality of wireless channels varies with time, frequency, and location, link adaptation can be used to improve transmission performance. With link adaptation, modulation order, coding rate, and transmit power can be selected according to *channel state information* (CSI). Traditional systems are built to operate on a fixed set of operating points [4], e.g., no power adaptation or rate adaptation. This results in excessive energy consumption or pessimistic data rate for peak channel conditions. Hence, a set of physical (PHY) layer parameters should be adjusted to adapt wireless channels to improve energy efficiency. Information theorists have studied energy-efficient transmission for at least two decades [5], [6]. The work in [5] defines reliable communication under a finite energy constraint in terms of the capacity per unit energy, which is the maximum number of bits that can be transmitted per unit energy. It is also shown in [5] that the capacity per unit energy is achieved using an unlimited number of degrees of freedom per information bit, e.g., with infinite bandwidth [7] or long-duration regime communications [8]. For example, the lowest order modulation should be always used while accommodating the delay constraint [8] to minimize energy consumption. The information-theoretic results derived in [7], [8] focus only on transmit power when considering energy consumption during transmission. In reality, a device also incurs additional circuit power during transmission, which is relatively independent of the transmission rate [9]–[11]. In this case, the method to transmit with the longest duration is no longer the best since circuit energy consumption is proportional to the transmission duration. The energy dissipation consisting of both transmitter electronics and *radio front* (RF) output is studied in [9], and several energy-minimization techniques, including modulation and multiple access protocols, are derived for short-range asymmetric micro-sensor systems. It is shown that a high order modulation may enable energy savings compared with binary modulation for some short-range applications by decreasing the transmission duration. In [10], these ideas are extended to a detailed energy consumption analysis specifically for both uncoded and coded *M-ary quadrature amplitude modulation* (M-QAM) and *multiple frequency shift keying* (MFSK) in additive white Gaussian noise channels. Here, energy-efficient transmission is formulated to find a tradeoff among transmission energy, circuit energy, and transmission duration. Similarly, a steepest descent gradient algorithm is designed in [12] to obtain the optimal rate that minimizes the average power consumption subject to a constraint on average throughput.

Due to limited wireless resources, intricate performance tradeoffs exist between an individual user and the whole network. The exploitation of diversity across all users will further reduce overall network energy consumption. Wireless resources can be managed in different domains to improve network energy efficiency. For example, in the time domain, e.g., in a *time-division multiple access* (TDMA) network, the channel medium is shared through time division. Each user tends to extend its transmission time to save energy, which contradicts the intention of energy savings of other users. Thus, the allocation of time duration among all users is critical in determining network energy efficiency. Because the modulation order determines data rate and thus the time transmitting a certain amount of information, finding the optimal slot length for each user is equivalent to determining its corresponding constellation size [13]. Spatial domain cooperation can also be used to improve system energy efficiency. It has been observed that significant energy savings can be achieved and the savings grow almost linearly with distance when either transmitter or receiver cooperation is allowed [14], [15].

While extensive efforts have been undertaken to improve energy-efficient resource management in both the spatial and time domains, little effort has been devoted to the frequency domain. In fact, increasing transmission bandwidth improves energy efficiency. However, it is impossible to allocate the entire system bandwidth exclusively to one user since this may hurt the energy efficiency of other users. Hence, it is critical to consider overall network energy efficiency when performing frequency-domain resource management. The frequency selectivity of wideband wireless channels further accentuates this necessity. Additionally, *orthogonal frequency division multiple access* (OFDMA) has emerged as one of the prime multiple access schemes for next generation wireless networks[16], [17]. While extensive research has been done to improve throughput [18], [19], limited has been conducted for energy-efficient communications in OFDMA systems. Our previous work in [11], [20]–[22] studied the uplink communications in wireless OFDMA systems to improve the energy efficiency of mobile users. Specifically, we focused on optimizing a "bits-per-Joule" metric to target energy-efficiency instead of throughput or peak rates. Circuit power consumption, in addition to transmit power, was also explicitly included in our optimization. In our previous work, the optimal solutions were obtained with the help of iterative approaches, which were not only complex but also incurred additional energy consumption due to multiple iterations. Therefore lower-complexity techniques for achieving energy-efficient communication are highly desirable.

In this paper, we will develop schemes to reduce the complexity associated with the iterative search techniques proposed in [11], [20], [21]. Both low-complexity energy-efficient link adaptation and resource allocation schemes will be designed. With the help of locally linear approximation, we use a time-averaged "bits-per-Joule" energy efficiency metric to obtain closed-form link adaptation and resource

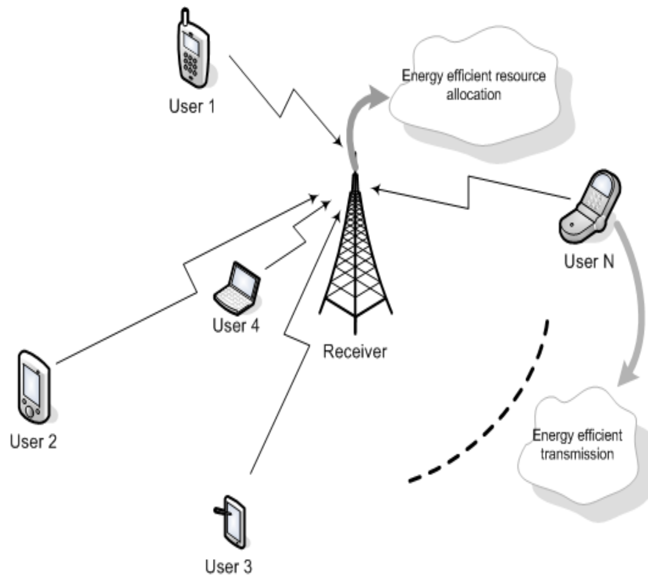

Fig. 1. Network architecture.

allocation schemes for uplink OFDMA systems in frequency-selective channels and the proposed approaches perform close to the global optimum. The rest of the paper is organized as follows. In Section II, we describe the system model and design objectives. Then we develop energy-efficient link adaptation and resource allocation schemes in Sections III and IV, respectively. Simulation results are provided in Section V. Finally, we conclude the paper and summarize the results in Section VI.

II. System Description

We focus on an uplink OFDMA system, as shown in Figure 1, as the *radio frequency* (RF) transmit power for a user dominates the limited power budget of a battery-constrained mobile device. The *base station* (BS) assigns subchannels for each user to optimize the overall network energy efficiency. Channels are assumed to be frequency-selective and with block fading, i.e. the channel state is constant within each frame [23]. Accurate channel state information is available to both BS and mobile users to optimize energy-efficient communications. The link adaptation and resource allocation settings are allowed to vary from one frame to another according to the channel state information.

Consider a network with N users and K subchannels. Denote the index set of all subchannels as $\mathcal{K} = \{1, 2, \cdots, K\}$ and the index set of subchannels assigned to User n at Frame t to be $\mathcal{C}_n[t]$. Each subchannel is assigned to only one user in each frame. Consequently,

$$\begin{aligned} \mathcal{C}_i[t] \bigcap \mathcal{C}_j[t] = \emptyset, \forall i \neq j \\ \bigcup_i \mathcal{C}_i[t] \subseteq \mathcal{K}, \end{aligned} \tag{1}$$

where $\emptyset$ denotes an empty set. The data rate of User n at Frame t is

$$r_n[t] = \sum_{k \in \mathcal{C}_n[t]} r_{nk}[t], \tag{2}$$

where $r_{nk}[t]$ is the data rate of User n at subchannel k and depends on the frequency-selective fading. If we use exponentially-weighted low-pass filter to get average data rate of User n at Frame t, it can be expressed as

$$T_n[t] = (1 - \frac{1}{w})T_n[t-1] + \frac{1}{w}r_n[t], \tag{3}$$

where $w \gg 1$.

Denote the *signal-to-noise ratio* (SNR) for reliable reception of $r_{nk}[t]$ to be

$$\eta_{nk} = S(r_{nk}[t]). \tag{4}$$

For example, the achievable data transmission rate r_i is determined by [24]

$$r_i = B\log_2(1 + \frac{\eta_i}{\Gamma}), \tag{5}$$

and correspondingly $S()$ is given by

$$\eta_{nk} = \left(2^{\frac{r_i}{B}} - 1\right)\Gamma, \tag{6}$$

where Γ is the SNR gap between the channel capacity and a practical coding and modulation scheme. For coded *quadrature-amplitude modulation* (QAM), the gap is [24]

$$\Gamma = 9.8 + \gamma_m - \gamma_c \text{ (dB)}, \tag{7}$$

where γ_m is system design margin and γ_c is coding gain. For Shannon capacity (P.373 of [25]), $\Gamma = 0$ dB. We can see in (6) that function $S(r)$ is strictly convex in r and $S(0) = 0$. Hence, in general, we do not specify the exact form of $S(r)$ and only assume $S(r)$ to be strictly convex in r and $S(0) = 0$. Denote the signal power attenuation of User n on Subchannel k at Frame t to be $g_{nk}[t]$, then the required power on Subchannel k for User n to transmit at a rate of $r_{nk}[t]$ will be

$$p_{nk}[t] = \frac{\eta_{nk}\sigma^2}{g_{nk}[t]} = \frac{S(r_{nk}[t])\sigma^2}{g_{nk}[t]}, \tag{8}$$

where σ^2 is the power of *additive white Gaussian noise* (AWGN). The overall transmit power of User n is

$$p_n[t] = \sum_{k \in \mathcal{C}_n[t]} p_{nk}[t]. \tag{9}$$

As indicated in [11], [20], circuit power, p_c, in addition to the transmit power, also needs to be considered in energy-efficient communications. While transmit power is used for reliable data transmission, circuit power represents energy consumption of device electronics themselves. The overall weighted moving average power consumption, $P_n[t]$, is also obtained using an exponentially weighted moving average low-pass filter, that is,

$$P_n[t] = (1 - \frac{1}{w})P_n[t-1] + \frac{1}{w}(p_n[t] + p_{cn}[t]). \tag{10}$$

The circuit power, $p_{cn}[t]$, is measured at Frame t by User n. Here, the power consumption of each user is divided into two parts. $p_{cn}[t]$ models the part of power consumption independent of the radio transmission, e.g. link adaptation, while $p_n[t]$ the part depending on the radio transmission.

For energy-efficient communications, users want to send as much data as possible with a given amount of energy. Hence, with energy $\triangle e$ consumed in a duration $\triangle t$, User n wants to send a maximum amount of data by choosing $r_{nk}[t], k \in \mathcal{C}_n[t]$, to maximize

$$\frac{T_n[t] \triangle t}{\triangle e}, \tag{11}$$

which is equivalent to maximize

$$u_n[t] = \frac{T_n[t]}{\triangle e / \triangle t} = \frac{T_n[t]}{P_n[t]}. \tag{12}$$

u_n is called average energy efficiency of User n. Adapting transmission rate and power to optimize equation (12) is referred to as energy-efficient link adaptation.

If the overall transmit power is fixed, the objective of Equation (12) is equivalent to maximizing the overall throughput and existing water-filling power allocation approaches [18], [25] can be used. However, besides adapting the power distributions on all subchannels, the overall transmit power can also be adapted according to the states of all subchannels and the history of data transmission and power consumption to maximize the average energy efficiency. Hence, the solution to Equation (12) is different from existing power allocation schemes that maximize throughput with power constraints.

When multiple users are involved in a wireless network, the BS determines subchannel assignment to optimize the overall network performance. Two multi-user energy efficiency metrics, arithmetic and geometric means of the energy efficiency of all users in the network, can be considered. Considering these performance metrics in the context of spectral efficiency, we note that the arithmetic-mean metric leads to power allocation for sum throughput maximization, and assures no fairness since some users may have zero throughput. However, the geometric-mean metric introduces proportional fairness among all users [26], [27]. Analogously, we call energy-efficiency optimization schemes using geometric- or arithmetic-mean metrics to be energy-efficient schedulers with or without fairness, respectively.

With the arithmetic-mean metric, the subchannels are allocated to maximize the arithmetic average of the energy efficiency of all users, i.e. to maximize

$$U[t] = \sum_{n=1}^{N} u_n[t]. \tag{13}$$

With the geometric-mean metric, the subchannels are allocated to maximize the geometric average of the energy efficiency of all users, i.e. to maximize

$$V[t] = \sum_{n=1}^{N} \log(u_n[t]). \tag{14}$$

Sometimes, the circuit power dominates the power consumption in the above optimization, e.g. in short-range communications where low transmit power is needed to compensate for path loss. For example, consider a commercial 802.11 network adapter, Cisco Aironet 802.11a/b/g Wireless CardBus Adapter. As shown in [28], its operating voltage is 3.3 Volts and when it transmits at 54 Mbps, the current is 554 milliamps. Then the overall power consumption, including both transmit power and circuit power, is $3.3 \times 554 = 1828$ milliwatts. However, as shown in [28], the transmit power for reliable data transmission can be only 20 milliwatts when the data rate is 54 Mbps for a communications range of 13 meters. Assuming a power amplifier efficiency of 20%, the overall transmit power for reliable data transmission for this device is expected to be 100 mW. Hence, the circuit power consumption is $1828 - 100 = 1728$ milliwatts. More examples can be easily found for other types of short-range communications, e.g. Bluetooth. In this case, maximizing energy efficiency in (12) is equivalent to maximizing throughput $T_n[t]$ as $P_n[t]$ is almost independent of transmit power allocation and rate adaptation. Correspondingly, (13) is equivalent to maximizing the sum of throughput weighted by the inverse of circuit power and (14) equals maximizing the product of throughput. The dependence of the optimization on circuit power will be further demonstrated later.

In the following, we develop link adaptation and resource allocation strategies in closed-forms, based on optimizing the energy-efficient metrics discussed in this section.

III. Energy-Efficient Link Adaptation

In this section, we investigate energy-efficient link adaptation for a single user with a given channel assignment. Therefore, user index, n, is dropped in the subsequent discussion in this section.

From Section II, we need to determine the data rates at all subchannels to maximize

$$\begin{aligned} u[t] =& \frac{T[t]}{P[t]} \\ =& \frac{(1-\frac{1}{w})T[t-1] + \frac{1}{w}\sum_k r_k[t]}{(1-\frac{1}{w})P[t-1] + \frac{1}{w}(\sum_k p_k[t] + p_c[t])}, \end{aligned} \tag{15}$$

where $p_k[t+1]$ is given by (8). Denote the data rate vector on all assigned subchannels to be $\mathbf{r}[t]$. Then $u[t]$ is a function of $\mathbf{r}[t]$. As shown in Appendix A, $u[t]$ is a strictly quasi-concave function of $\mathbf{r}[t]$ and hence, a unique globally optimal rate vector, $\mathbf{r}^*[t]$, always exists [29] and every element in $\mathbf{r}^*[t]$ satisfies

$$\frac{\partial u[t]}{\partial r_k[t]} = 0 \tag{16}$$

if $r_k[t] > 0$. Note that in (15), only $r_k[t]$ and $p_k[t]$ are functions of $r_k[t]$ and other terms are independent of $r_k[t]$. Then solving (16) yields the following optimal rate condition

$$\frac{\partial p_k[t]}{\partial r_k[t]} = \frac{P[t]}{T[t]} = \frac{1}{u[t]}, \forall k. \tag{17}$$

If $w \gg 1$, as assumed, $P[t] \approx P[t-1]$ and $T[t] \approx T[t-1]$,

$$\frac{\partial p_k[t]}{\partial r_k[t]} = \frac{P[t-1]}{T[t-1]} = \frac{1}{u[t-1]}, \forall k. \tag{18}$$

Together with (8), we have

$$S^{'}(r_k[t]) = \frac{1}{u[t-1]}\frac{g_k[t]}{\sigma^2}, \forall k. \tag{19}$$

where $S^{'}(\cdot)$ is the derivative of the function $S(\cdot)$. Consequently, the optimal data rate follows immediately,

$$r_k^*[t] = \max\left(S^{'-1}(\frac{1}{u[t-1]}\frac{g_k[t]}{\sigma^2}), 0\right) \forall k \in \mathcal{C}[t]. \tag{20}$$

where $S^{'-1}()$ is the inverse function of $S^{'}$. The corresponding optimal power allocation is

$$p_k^*[t] = \frac{S(r_k^*[t])\sigma^2}{g_k[t]}, \forall k \in \mathcal{C}[t]. \tag{21}$$

Note that in the above derivation, approximation is only used in (18). If w is sufficiently large, the approximation error is closed to zero and (20) and (21) are almost globally optimal.

If each subchannel is experiencing AWGN and the Shannon capacity (P.373 of [25]) is achieved on each subchannel, $r = B\log_2(1+\eta)$. Then $S(r) = 2^{\frac{r}{B}} - 1$, where B is the subchannel bandwidth. The optimal data rate on Subchannel n is

$$r_k^*[t] = \max\left(B\log_2\left(\frac{Bg_k[t]}{u[t-1]\sigma^2\log 2}\right), 0\right) \forall k \in \mathcal{C}[t]. \tag{22}$$

The corresponding optimal power allocation on Subchannel n is

$$p_k^*[t] = \max\left(\frac{B}{u[t-1]\log 2} - \frac{\sigma^2}{g_k[t]}, 0\right) \forall k \in \mathcal{C}[t], \tag{23}$$

which is a water-filling power allocation with a water level of $\frac{B}{u[t-1]\log 2}$, as in Figure 2. We can see that the energy-efficient link adaptation in (20), (21), (22), and (23) is determined by $u[t-1]$ and $g_k[t]$, and is expressed in closed form. This significantly reduces the complexity associated with the iterative solutions developed earlier in [11]. From Figure 2, we can also see that every shadowed part corresponds to the power allocated on each subchannel.

IV. Energy-Efficient Resource Allocation

In this section, we will consider low-complexity energy-efficient resource allocation for multi-user networks. Here index n is necessary to indicate a particular user. Schedulers based on both the arithmetic and the geometric mean will be derived.

A. Energy-Efficient Scheduler without Fairness Consideration

In this section, the subchannels are assigned such that the sum energy efficiency $U[t]$ is maximized. Since $U[t-1]$ is fixed, it is equivalent to maximize

$$\begin{aligned} \triangle U &= U[t] - U[t-1] \\ &= \sum_{n=1}^{N} u_n[t] - \sum_{n=1}^{N} u_n[t-1] \\ &= \sum_{n=1}^{N}(u_n[t] - u_n[t-1]). \end{aligned} \tag{24}$$

We can see that

$$\begin{aligned} u_n[t] - u_n[t-1] &= \frac{T_n[t]}{P_n[t]} - \frac{T_n[t-1]}{P_n[t-1]} \\ &= \frac{T_n[t]P_n[t-1] - P_n[t]T_n[t-1]}{P_n[t]P_n[t-1]}. \end{aligned} \tag{25}$$

Substituting Equations (3) and (10) into (25), we have

$$u_n[t] - u_n[t-1]$$
$$= \left(P_n[t-1]\sum_{k\in\mathcal{C}_n[t]} r_{nk}[t] - \right.$$

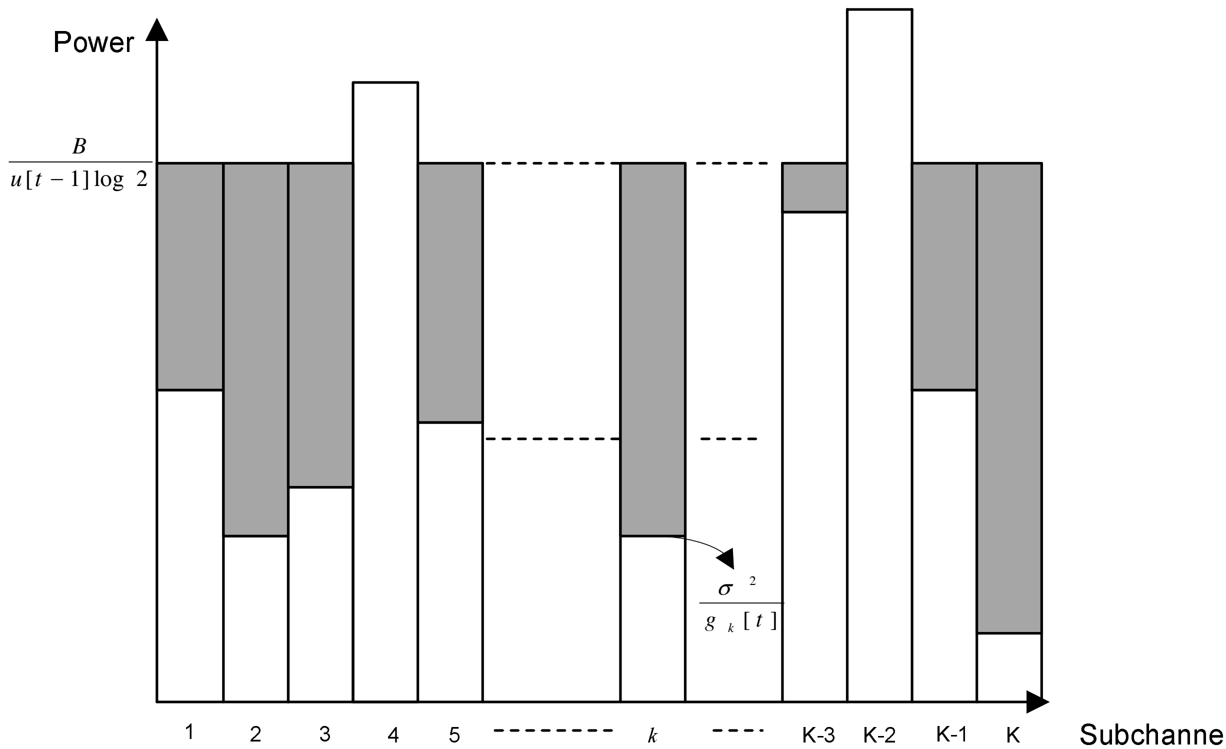

Fig. 2. Low-complexity energy-efficient water-filling power allocation.

$$
\begin{aligned}
&T_n[t-1](\sum_{k\in\mathcal{C}_n[t]} p_{nk}[t]+p_{cn}[t])\Bigg)/(wP_n[t]P_n[t-1]) \\
&=\sum_{k\in\mathcal{C}_n[t]} \frac{P_n[t-1]r_{nk}[t]-T_n[t-1]p_{nk}[t]}{wP_n[t]P_n[t-1]} - \frac{T_n[t-1]p_{cn}[t]}{wP_n[t]P_n[t-1]} \\
&=\sum_{k=1}^{K} I_k(\mathcal{C}_n[t])\frac{P_n[t]r_{nk}[t]-T_n[t-1]p_{nk}[t]}{wP_n[t]P_n[t-1]} \\
&\quad - \frac{T_n[t-1]p_{cn}[t]}{wP_n[t]P_n[t-1]},
\end{aligned}
$$

where indicator function $I_k(\mathcal{C}_n)$ is defined as

$$
I_k(\mathcal{C}_n) = \begin{cases} 1 & k\in\mathcal{C}_n, \\ 0 & \text{otherwise.} \end{cases} \tag{26}
$$

Hence, the subchannel assignment is to maximize

$$
\begin{aligned}
\triangle U &= \sum_{n=1}^{N}(u_n[t]-u_n[t-1]) \\
&= \sum_{n=1}^{N}\sum_{k=1}^{K} I_k(\mathcal{C}_n[t])\frac{P_n[t-1]r_{nk}[t]-T_n[t-1]p_{nk}[t]}{wP_n[t]P_n[t-1]} \\
&\quad - \sum_{n=1}^{N}\frac{T_n[t-1]p_{cn}[t]}{wP_n[t]P_n[t-1]} \\
&= \sum_{k=1}^{K}\sum_{n=1}^{N} I_k(\mathcal{C}_n[t])\frac{P_n[t-1]r_{nk}[t]-T_n[t-1]p_{nk}[t]}{wP_n[t]P_n[t-1]} \\
&\quad - \sum_{n=1}^{N}\frac{T_n[t-1]p_{cn}[t]}{wP_n[t]P_n[t-1]}.
\end{aligned}
$$

Denote the allocation metric to be

$$
\begin{aligned}
J(n,k) &= \frac{P_n[t-1]r_{nk}[t]-T_n[t-1]p_{nk}[t]}{P_n[t]P_n[t-1]} \\
&\approx \frac{P_n[t-1]r_{nk}[t]-T_n[t-1]p_{nk}[t]}{P_n^2[t-1]} \\
&= \frac{r_{nk}[t]}{P_n[t-1]} - u_n[t-1]\frac{p_{nk}[t]}{P_n[t-1]},
\end{aligned} \tag{27}
$$

where $r_{nk}[t]$ is given by (20) and $p_{nk}[t]$ (21).

It is easy to see that $\triangle U$ is maximized by assigning subchannel k to the user with the highest allocation metric $J(n,k)$ on that subchannel, that is, the optimal subchannel assignment is

$$
\mathcal{C}_n^* = \{k|J(n,k) > J(m,k), \forall m \neq n\}, \forall n. \tag{28}
$$

Note that in the above derivation, approximation is used in (27). If w is sufficiently large, the approximation error is close to zero and the proposed scheduler is almost globally optimal.

When the circuit power dominates the power consumption, the allocation metric is

$$
J_t(n,k) \approx \frac{r_{nk}[t]}{P_n[t-1]}. \tag{29}
$$

Assume all users consume the same circuit power and $P_n[t-1]$ is the same for all users. Since the user with the maximum $r_{nk}[t]$ is the same as the one with the maximum SINR on that subchannel, the energy-efficient scheduler is equivalent to applying the traditional max-SINR scheduler on each subchannel to achieve the highest spectral efficiency [30], which is,

$$
\mathcal{C}_n^* = \{k|r_{n,k} > r_{m,k}, \forall m \neq n\}, \forall n. \tag{30}
$$

B. Energy-Efficient Scheduler with Fairness Consideration

In order to maximize the geometric mean of the energy efficiency of all users, the subchannels are assigned to maximize

$$V[t] = \sum_{n=1}^{N} \log(u_n[t]), \tag{31}$$

which is equivalent to maximize

$$\begin{aligned} \triangle V &= V[t] - V[t-1] \\ &= \sum_{n=1}^{N} \log(u_n[t]) - \sum_{n=1}^{N} \log(u_n[t-1]) \\ &= \sum_{n=1}^{N} \left(\log\left(\frac{T_n[t]}{T_n[t-1]}\right) - \log\left(\frac{P_n[t]}{P_n[t-1]}\right)\right). \end{aligned} \tag{32}$$

Using the Taylor series expansion and the fact that $w \gg 1$, we have

$$\begin{aligned} \log\left(\frac{T_n[t]}{T_n[t-1]}\right) &= \log\left(1 - \frac{1}{w} + \frac{\frac{1}{w}\sum_{k\in\mathcal{C}_n} r_{nk}[t]}{T_n[t-1]}\right) \\ &\approx \log(1-\frac{1}{w}) + \frac{\sum_{k\in\mathcal{C}_n} r_{nk}[t]}{T_n[t-1](w-1)}. \end{aligned} \tag{33}$$

Similarly, we have

$$\begin{aligned} &\log\left(\frac{P_n[t]}{P_n[t-1]}\right) \\ &\approx \log(1-\frac{1}{w}) + \frac{\sum_{k\in\mathcal{C}_n} p_{nk}[t] + p_{cn}[t]}{P_n[t-1](w-1)}. \end{aligned} \tag{34}$$

Hence, $\triangle V$ can be expressed as

$$\begin{aligned} \triangle V &= \sum_{n=1}^{N} \left(\frac{\sum_{k\in\mathcal{C}_n[t]} r_{nk}[t]}{T_n[t-1](w-1)} \right. \\ &\quad \left. - \frac{\sum_{k\in\mathcal{C}_n[t]} p_{nk}[t] + p_{cn}[t]}{P_n[t-1](w-1)} \right) \\ &= \sum_{n=1}^{N}\sum_{k=1}^{K} \left(I_k(\mathcal{C}_n[t]) (\frac{r_{nk}[t]}{T_n[t-1](w-1)} \right. \\ &\quad \left. - \frac{p_{nk}[t]}{P_n[t-1](w-1)}) \right) - \sum_{n=1}^{N} \frac{p_{cn}[t]}{P_n[t-1](w-1)} \\ &= \sum_{k=1}^{K}\sum_{n=1}^{N} \left(I_k(\mathcal{C}_n[t]) (\frac{r_{nk}[t]}{T_n[t-1]} \right. \\ &\quad \left. - \frac{p_{nk}[t]}{P_n[t-1]})/(w-1) \right) - \sum_{n=1}^{N} \frac{p_{cn}[t]}{P_n[t-1](w-1)}. \end{aligned}$$

Denote the allocation metric to be

$$J_f(n,k) = \frac{r_{nk}[t]}{T_n[t-1]} - \frac{p_{nk}[t]}{P_n[t-1]}, \tag{35}$$

where $r_{nk}[t]$ is given by (20) and $p_{nk}[t]$ (21). Therefore, $\triangle V$ is maximized by assigning subchannel k to the user with the highest allocation metric $J_f(n,k)$ on that subchannel, that is, the optimal subchannel assignment achieving proportional fairness is

$$\mathcal{C}_n^* = \{k | J_f(n,k) > J_f(m,k), \forall m \neq n\}, \forall n. \tag{36}$$

In the above derivation, approximations are used in (33) and (34). If w is sufficiently large, the approximation error is zero and the proposed scheduler is almost globally optimal.

When the circuit power dominates the power consumption, the allocation metric is

$$J_{tf}(n,k) \approx \frac{r_{nk}[t]}{T_n[t-1]}, \tag{37}$$

and the energy-efficient scheduler is equivalent to applying the traditional proportional-fair scheduler [26], [27] on each subchannel, that is,

$$\mathcal{C}_n^* = \{k | J_{tf}(n,k) > J_{tf}(m,k), \forall m \neq n\}, \forall n. \tag{38}$$

V. Numerical Results

In the previous sections, we have obtained closed-form expressions for energy-efficient link adaptation and resource allocation. In this section, we compare these proposed schemes with the global optima to evaluate the suboptimality gap. For link adaptation, the global optimum is the solution that globally maximizes (15). Since $u[t]$ in (15) is strictly quasi-concave in the data rate vector $\mathbf{r}[t]$, a local optimal solution is also globally optimal [29] and we use the iterative approach in [21] to obtain the global optimal link adaptation. For energy-efficient schedulers, the global optimum is the solution that globally maximizes $U[t]$ in (13) or $V[t]$ in (14). We exhaustively search all possible subchannel assignments as well as the corresponding optimal link adaptation. The solution that achieves the highest $U[t]$ or $V[t]$ is globally optimal. The weight of the exponentially weighted low-pass filter, the number of subchannels, and the number of users in the system determine the approximation accuracy in deriving the closed-form approaches in this paper. Hence, we will focus on their impact on the system energy efficiency performance. Although actual throughput and energy efficiency trade offs are important part of energy aware design, they have been discussed in our previous work [21], [22] and are not repeated here. The focus here is to evaluate the performance of low-complexity schemes.

ITU multipath pedestrian channel A [31] is used to model the frequency-selective fading. Capacity approaching coding is assumed. Figure 3 compares the low-complexity suboptimal approaches with the global optimal approaches for energy-efficient link adaptation when there are 10 subchannels in the system. The energy efficiency of the proposed link adaptation is normalized by the energy efficiency of the global optimal solution. We show the normalized energy efficiency with different weights, w. ϵ is the transmit power to circuit power ratio assuming the transmit power is allocated such that the average achieved spectral efficiency is one bit/s/Hz. The circuit power is varied to have different ϵ values. From the figure the proposed link adaptation performs closely to the global optimum, with a performance loss of less than 2% when $w > 10$. Figure 4 further demonstrates the normalized energy efficiency when the system has different numbers of subchannels. We can see while more subchannels result in less approximation accuracy of the proposed low-complexity energy-efficient link adaptation scheme, larger w can be used to ensure a small suboptimality gap.

Figure 5 shows the normalized energy efficiency of different schedulers in a three-user network. Since we need to exhaustively search all possible subchannel assignments as well as the corresponding optimal link adaptation to find the global optimal solution, the complexity grows exponentially. For example, when there are three users and eight subchannels in the network, there are $3^8 = 6561$ possible channel assignments and for each channel assignment we need to iteratively search for the optimal link adaptation for each user. To reduce the complexity, the network is configured eight subchannels. As shown in Figure 5, the performance loss of the proposed low-complexity close-form schedulers and link adaptation is within 5% when $w > 10$. Table I further demonstrates the impact of the number of users on the system performance when the network has eight subchannels.

As shown in Figures 3 and 5, both weight, w, and transmit to circuit power ratio, ϵ, impact the system performance. The selection of the weight determines the approximation accuracy of (18), (27), (33), and (34). For example, in (27), $P_n[t]$ is approximated by $P_n[t-1]$. It is easy to see that when $w = \infty$, $P_n[t] = P_n[t-1]$ according to (10) and there is no approximation error. However, whenever w is finite, there is an approximation error and this error decreases with w. This can be seen if we define the error to be $err = |P_n[t] - P_n[t-1]|$, in which $P_n[t]$ is the true value and $P_n[t-1]$ is the approximate value of $P_n[t]$ in (27). Referring to (10), $err = \frac{|-P_n[t-1]+p_n[t]+p_{cn}[t]|}{w}$ and err decreases with w. Furthermore, the approximation error determines the accuracy of the allocation metric (third line of (27)). Hence, the selection of w impacts the system performance. Similar analysis can be applied to (18), (33) and (34). The impact of the transmit power to circuit power ratio, ϵ, is the same as the impact of circuit power. When circuit power is larger, our previous work [21] has shown that energy-efficient link adaptation will choose higher data rate. Intuitively, this is because when circuit power is larger, energy-efficient link adaptation tends to transmit at a higher data rate to reduce the transmission time such that the circuit energy consumption can be reduced. Hence, when circuit power is larger, the actual optimal energy-efficient data rate is also larger. Furthermore, the optimal transmit power is also larger according to (21). However, to obtain closed-form solutions, we have approximated that $T[t] \approx T[t-1]$ and $P[t] \approx P[t-1]$. Given w, the approximation error depends on the actual optimal data rate and transmit power in $T[t]$ and $P[t]$. Hence, the circuit power impacts the approximation accuracy and thus the system performance. Besides w and ϵ, the number of users and sub-channels in the system also impact the system performance. Note that approximation is used on each subchannel for each user to achieve close-form link adaptation on all subchannels. When there are multiple users and subchannels in the system, the approximation errors on all subchannels of all users will be accumulated to impact the behavior of both scheduling and link adaptation. Hence, the suboptimality gap also depends on the number of users and subchannels in the system, as shown in Table I and Figure 4. However, the approximation accuracy can always be improved by using higher values of w.

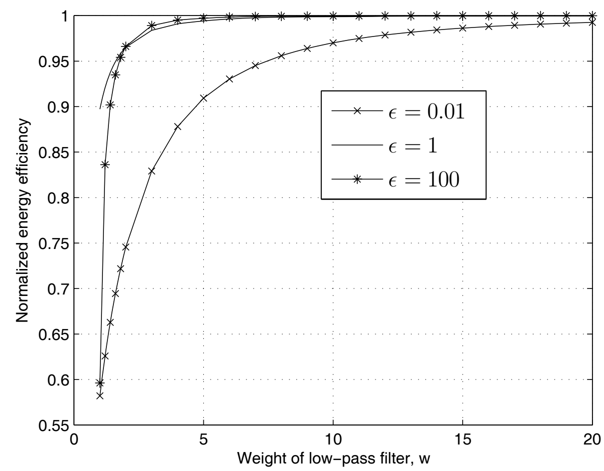

Fig. 3. Normalized energy efficiency of a single link varying the weight of low-pass filer, w (ϵ: transmit to circuit power ratio).

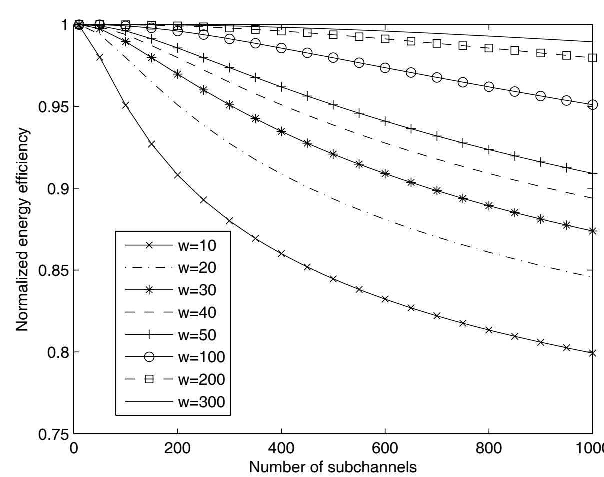

Fig. 4. Normalized energy efficiency of a single link varying the number of subchannels ($\epsilon = 1$).

VI. Conclusion

This paper developed low-complexity energy efficient link adaptation and resource allocation schemes for uplink OFDMA communication systems. By considering time averaged bit-per-Joule metrics, we derived energy optimal link adaptation and resource scheduling techniques in closed forms and the results are also summarized in Table II. Our solutions are applicable for frequency selective channels and account for time varying circuit power in computing the time-averaged energy efficient metrics. The simulation results included in this paper show that the proposed low-complexity schemes perform close to the globally optimum solutions obtained through exhaustive search, under a variety of scenarios.

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TABLE I
IMPACT OF THE NUMBER OF USERS

Number of Users	Normalized energy efficiency of Max-AM scheduler	Normalized energy efficiency of Max-GM scheduler
2	0.958	0.9952
3	0.9394	0.9931
4	0.9337	0.9926

TABLE II
MAIN RESULTS

Function	Formula
Link adaptation	$r_k^*[t] = \max\Big(S'^{-1}\big(\frac{1}{u[t-1]}\frac{g_k[t]}{\sigma^2}\big), 0\Big)$ and $p_k^*[t] = \frac{S(r_k^*[t])\sigma^2}{g_k[t]}, \forall k \in \mathcal{C}[t]$.
Scheduler w/o fairness	$J(n,k) = \frac{r_{nk}[t]}{P_n[t-1]} - u_n[t-1]\frac{p_{nk}[t]}{P_n[t-1]}$ and $\mathcal{C}_n^* = \{k \vert J(n,k) > J(m,k), \forall m \neq n\}, \forall n$
Scheduler w/ fairness	$J_f(n,k) = \frac{r_{nk}[t]}{T_n[t-1]} - \frac{p_{nk}[t]}{P_n[t-1]}$ and $\mathcal{C}_n^* = \{k \vert J_f(n,k) > J_f(m,k), \forall m \neq n\}, \forall n$

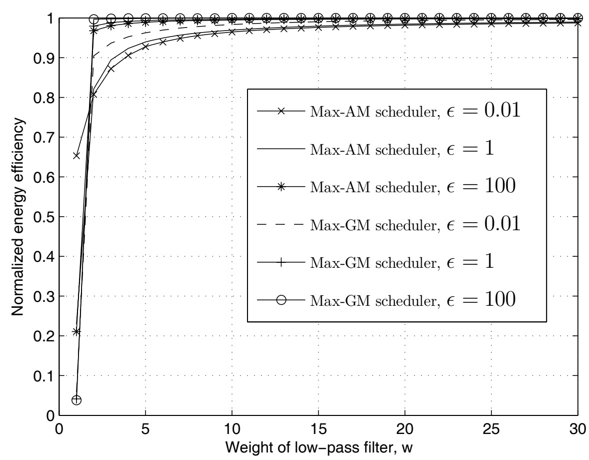


Fig. 5. Normalized average energy efficiency of a three-user network (w: weight of low-pass filer; ϵ: the average transmit power to circuit power ratio; Max-AM: max arithmetic mean energy-efficient scheduler; and Max-GM: max geometric mean energy-efficient scheduler).

APPENDIX A
QUASICONCAVITY OF ENERGY EFFICIENCY FUNCTION

Proof: Denote the sublevel sets of $u[t]$ as

$$\Gamma_\alpha = \{\mathbf{r}[t] \succeq \mathbf{0} \big\vert u[t] \geq \alpha\} \qquad \text{for any real } \alpha, \tag{A.39}$$

where $\mathbf{0}$ is the zero vector and symbol $\succeq$ denotes vector inequality and $\mathbf{r}[t] \succeq \mathbf{0}$ means each element of $\mathbf{r}[t]$ is nonnegative. According to Proposition C.9 of [29], $u[t]$ is strictly quasiconcave if and only if Γ_α is strictly convex for any real number α. When $\alpha \leq 0$, $u[t] \geq \alpha$ for all $\mathbf{r}[t]$. Hence, Γ_α is strictly convex when $\alpha \leq 0$. Now we investigate the case when $\alpha > 0$. Γ_α is equivalent to

$$\begin{aligned}\Gamma_\alpha &= \left\{\mathbf{r}[t] \succeq \mathbf{0} \,\middle\vert\, \frac{(1-\frac{1}{w})T[t-1] + \frac{1}{w}\sum_k r_k[t]}{(1-\frac{1}{w})P[t-1] + \frac{1}{w}(\sum_k p_k[t] + p_c[t])} \geq \alpha\right\} \\ &= \left\{\mathbf{r}[t] \succeq \mathbf{0} \,\middle\vert\, \frac{1}{w}\sum_k r_k[t] - \frac{\alpha}{w}\sum_k p_k[t] + Q \geq 0\right\} \\ &= \left\{\mathbf{r}[t] \succeq \mathbf{0} \,\middle\vert\, \frac{1}{w}\sum_k r_k[t] - \frac{\alpha}{w}\sum_k \frac{S(r_k[t])\sigma^2}{g_k[t]} + Q \geq 0\right\}.\end{aligned} \tag{A.40}$$

where $Q = (1-\frac{1}{w})T[t-1] - \alpha(1-\frac{1}{w})P[t-1] - \frac{\alpha}{w}p_c[t]$. Since $S(r)$ is strictly convex in r, it is easy to see that Γ_α is strictly convex. Hence, we have the strict quasiconcavity of $u[t]$. ■

REFERENCES

[1] G. W. Miao, N. Himayat, Y. (G.) Li, and S. Talwa, "Low-complexity energy-efficient OFDMA," in *Proc. 2009 IEEE ICC*, pp. 1–5.
[2] K. Lahiri, A. Raghunathan, S. Dey, and D. Panigrahi, "Battery-driven system design: a new frontier in low power design," in *Proc. 2002 Intl. Conf. VLSI Design*, pp. 261–267.
[3] G. W. Miao, N. Himayat, Y. Li, and A. Swami, "Cross-layer optimization for energy-efficient wireless communications: a survey," *Wireless Commun. Mobile Comput.*, vol. 9, no. 4, pp. 529–542, Apr. 2009.
[4] Atheros Communications, "White paper: power consumption & energy efficiency," 2003.
[5] R. G. Gallager, "Power limited channels: coding, multiaccess, and spread spectrum," in *Proc. 1988 Conf. Inf. Sci. Syst.*, vol. 1.
[6] S. Verdu, "On channel capacity per unit cost," *IEEE Trans. Inf. Theory.*, vol. 36, no. 5, pp. 1019–1030, Sep. 1990.
[7] ——, "Spectral efficiency in the wideband regime," *IEEE Trans. Inf. Theory.*, vol. 48, no. 6, pp. 1319–1343, June 2002.
[8] F. Meshkati, H. V. Poor, S. C. Schwartz, and N. B. Mandayam, "An energy-efficient approach to power control and receiver design in wireless networks," *IEEE Trans. Commun.*, vol. 5, no. 1, pp. 3306–3315, Nov. 2006.
[9] A. Y. Wang, S. Chao, C. G. Sodini, and A. P. Chandrakasan, "Energy efficient modulation and MAC for asymmetric RF microsensor system," in *2001 Int. Symp. Low Power Electron. Design*, pp. 106–111.
[10] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2349–2360, Sep. 2005.
[11] G. W. Miao, N. Himayat, Y. Li, and D. Bormann, "Energy-efficient design in wireless OFDMA," in *Proc. 2008 IEEE ICC*, pp. 3307–3312.
[12] C. Schurgers and M. B. Srivastava, "Energy optimal scheduling under average throughput constraint communications," in *Proc. 2003 IEEE Conf. Commun.*, vol. 3, pp. 1648–1652.
[13] S. Cui, A. Goldsmith, and A. Bahai, "Joint modulation and multiple access optimization under energy constraints," in *Proc. 2004 IEEE GLOBECOM*, vol. 1, pp. 151–155.
[14] ——, "Energy-efficiency of MIMO and cooperative MIMO techniques in sensor networks," in *IEEE J. Sel. Areas Commun.*, vol. 22, no. 6, pp. 1089–1098, Aug. 2004.
[15] S. K. Jayaweera, "An energy-efficient virtual MIMO architecture based on V-BLAST processing for distributed wireless sensor networks," in *Proc. 2004 IEEE SECON*, pp. 299–308.
[16] Draft IEEE std 802.16e/D9, "IEEE standard for local and metropolitan area networks part 16: air interface for fixed and mobile broadband wireless access systems," June 2005.
[17] 3GPP, TR 25.913, "Requirements for evolved UTRA and evolved UTRAN." Available: www.3gpp.org.
[18] Y. (G.) Li and G. L. Stüber, *OFDM for Wireless Communications*. Springer, 2006.

[19] G. W. Miao and Z. Niu, "Practical feedback design based OFDM link adaptive communications over frequency selective channels," in *Proc. 2006 IEEE ICC*, pp. 4624–4629.

[20] G. Miao, N. Himayat, and Y. Li, "Energy-efficient transmission in frequency-selective channels," in *Proc. 2008 IEEE Globecom*, pp. 1–5.

[21] ——, "Energy-efficient link adaptation in frequency-selective channels," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 545–554, Feb. 2010.

[22] G. Miao, N. Himayat, Y. Li, A. T. Koc, and S. Talwar, "Interference-aware energy-efficient power optimization," in *Proc. 2009 IEEE ICC*, pp. 1–5.

[23] R. J. McEliece and W. E. Stark, "Channels with block interference," *IEEE Trans. Inf. Theory.*, vol. 30, no. 1, pp. 44-53, Jan. 1984.

[24] J. M. Cioffi, *A Multicarrier Primer*. ANSI T1E1, 1999.

[25] R. G. Gallager, *Information Theory and Relaible Communication*. John Wiley & Sons, Inc., 1968.

[26] R. Mazumdar, L. G. Mason, and C. Douligieris, "Fairness in network optimal flow control: optimality of product forms," *IEEE Trans. Commun.*, vol. 39, pp. 775–782, May 1991.

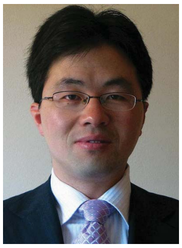
[27] P. Viswanath, D. N. C. Tse, and R. L. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory.*, vol. 48, pp. 1277–1294, June 2002.

[28] Cisco, "Cisco aironet 802.11a/b/g wireless cardbus adapter data sheet." Available: http://www.cisco.com/en/US/prod/collateral/wireless/ps6442/ps4555/ps5818/product_data_sheet09186a00801ebc29.pdf

[29] E. Wolfstetter, *Topics in Microeconomics: Industrial Organization, Auctions, and Incentives*. Cambridge University Press, 1999.

[30] G. Song and Y. (G.) Li, "Cross-layer optimization for OFDM wireless networks—part I: theoretical framework," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 614–624, 2005.

[31] ITU-R Recommendation M.1225, "Guidelines for evaluation of radio transmission technologies for IMT-2000," 1997.



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